

Precision cosmology and the landscape*

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Precision cosmology and the landscape

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ABSTRACT: After reviewing the cosmological constant problem—why is Λ not huge?—I outline the two basic approaches that had emerged by the late 1980s, and note that each made a clear prediction. Precision cosmological experiments now indicate that the cosmological constant is nonzero. This result strongly favors the environmental approach, in which vacuum energy can vary discretely among widely separated regions in the universe. The need to explain this variation from first principles constitutes an observational constraint on fundamental theory. I review arguments that string theory satisfies this constraint, as it contains a dense discretuum of metastable vacua. The enormous landscape of vacua calls for novel, statistical methods of deriving predictions, and it prompts us to reexamine our description of spacetime on the largest scales. I discuss the effects of cosmological dynamics, and I speculate that weighting vacua by their entropy production may allow for prior-free predictions that do not resort to explicitly anthropic arguments.

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1. Introduction

The quest for quantum gravity is driven by a desire for consistency and unity of physical law. Quantum mechanics and the general theory of relativity are hard to fit under one roof. String theory succeeds at this task, exhibiting a level of mathematical rigor and richness of structure that has yet to be matched by other approaches.

Unfortunately, the subject has been lacking guidance from experiment. Particle accelerators, in particular, are unlikely to probe effects of quantum gravity directly. The energies that can be attained are many orders of magnitude too low. This problem has nothing to do with string theory. It arises the minute we turn our attention to quantum gravity, because gravity is extremely weak in scattering experiments.

On large scales, however, gravity rules. The expansion of the universe dominates over all other dynamics at distances above 100 Mpc. Similarly, once matter condenses enough to form a black hole, no known force can prevent its total collapse into a singularity. In the early universe, moreover, quantum effects can be important. Perhaps, then, string theory should be looking towards cosmology for guidance.

In fact, recent years have seen a remarkable transformation. String theory has become driven, to a significant extent, by the results of precision experiments in cosmology. The discovery of dark energy [1, 2] suggests that vacuum energy is an environmental variable. String theory naturally provides for variability of the cosmological constant, with a fine enough spacing to accommodate the observed value [3]. In this sense, recent cosmological observations constitute observational evidence for the theory. Moreover, they have focussed attention on the large number of metastable vacua—the string landscape [3–5]—believed to be responsible for this variability.¹

Before presenting conclusions from precision experiments, I will argue in Sec. 2 that much can be learned from far more primitive observations of the cosmos. A simple question—why is the universe so large?—translates into a number of major challenges to theoretical cosmology. One, the flatness problem, motivated the theory of inflation, which went on to explain the origin of structure in the universe, making a number of specific predictions. Another, the cosmological constant problem, is especially closely related to fundamental theory: Why is the energy of empty space more than 120 orders of magnitude smaller than predicted by quantum field theory?

Most early discussions of the cosmological constant problem tended to embrace one of two distinct approaches. Either the cosmological constant has to be zero due to some unknown symmetry; or it is an environmental variable that can vary over distances large compared to the visible universe, and observers can only live in regions where it is anomalously small. Though neither of these approaches had been developed into concrete models, each made a signature prediction: that the cosmological constant is zero, or that it is small but nonzero.

The refined experiments of the last ten years have amassed additional evidence for inflation, and they have managed to discriminate clearly between the two approaches to the cosmological constant problem. The discovery of nonzero dark energy, in particular,

¹For a detailed review and extensive references, see Ref. [6]. For a less technical discussion of the issues covered in the present article, see Ref. [7].

is precisely what the second, environmental approach predicted, and it all but rules out the first approach. These results, summarized in Sec. 3, are the empirical foundation of the landscape of string theory.

In Sec. 4, I will describe a concrete model that realizes the second approach in string theory. The topological complexity of compact extra dimensions leads to an exponentially large potential landscape. Its metastable vacua form a dense “discretuum” of values of the cosmological constant. Every vacuum will be realized in separate regions, each bigger than the visible universe, but structure (and thus observers) form only in those regions where the cosmological constant is sufficiently small.

In Sec. 5, I will discuss some of the novel challenges posed by the string landscape. The greatest challenge, perhaps, is to develop methods for making predictions in a theory with 10^{500} metastable vacua. In fact, this difficulty is sometimes presented as insurmountable, but I will argue that it just comes down to a lot of hard work. In particular, I will argue that the correct statistical treatment of vacua necessitates a departure from the traditional, global description of spacetime. I will further propose a statistical weighting of vacua based on entropy production, which performs well in comparison with far more specific anthropic conditions. A general weighting of this type may pave the way for a calculation of the size of the universe from first principles.

2. Why is the universe large?

In cosmology, the most naive questions can be the most profound. A famous example is Olbers’ paradox: Why is the sky dark at night? In this spirit, let us ask why the universe is large. To quantify “large”, recall that only a single length scale can be constructed from the known constants of nature: the Planck length

$$l_P = \sqrt{\frac{G\hbar}{c^3}} \approx 1.616 \times 10^{-33} \text{cm} . \quad (2.1)$$

Here G denotes Newton’s constant and c is the speed of light.²

The actual size of the universe is larger than this fundamental length by a factor

$$H^{-1} = .8 \times 10^{61} . \quad (2.2)$$

Here $H \approx 70 \text{ km/s/Mpc}$ is the Hubble scale, and H^{-1} is the Hubble length. Of course, this refers to the size of the universe as we see it today, and thus is only a lower bound on the length scales that may characterize the universe as a whole.³

²In the remainder I will work mostly in Planck units. For example, $t_P = l_P/c \approx .539 \times 10^{-43} \text{s}$ and $M_P = 2.177 \times 10^{-5} \text{g}$.

³As I shall discuss below, there is evidence that the universe is exponentially larger than the visible universe, but that we will never see a region larger than $.98 \times 10^{61}$, no matter how long we wait.

The dynamical behavior of a system usually reflects the scales of the input parameters, and other scales constructed from them by dimensional analysis. For example, the ground state of a harmonic oscillator of mass m and frequency ω has a position uncertainty of order $(m\omega)^{-1/2}$. The parameters entering cosmology are G , \hbar , and c , so l_P is the natural length scale obtained by dimensional analysis. Thus, Eq. (2.2) represents an enormous hierarchy of scales. Where does this large number come from?

At the very end of this article, I will speculate about the origin of the number 10^{61} . For now, let us simply consider the qualitative fact that the universe is large compared to the Planck scale—a fact that is plain to the naked eye, no precision experiments required. We will see that some of the most famous problems in theoretical cosmology are tied to this basic observation: the flatness problem, and the cosmological constant problem.

2.1 The flatness problem and inflation

We live in a universe that is spatially isotropic and homogeneous on sufficiently large scales. The spatial curvature is constant, and it is remarkably small. By the Einstein equation, this can be related to the statement that the average density ρ is not far from the critical density,

$$\Omega \equiv \frac{\rho}{\rho_c} \sim O(1) , \quad (2.3)$$

where

$$\rho_c = \frac{3H^2}{8\pi} . \quad (2.4)$$

This is surprising because it means that the early universe was flat to fantastic accuracy. Through much of the history of the universe, Ω has been pushed away from 1. Einstein's equation implies that

$$|\Omega - 1| = (\dot{a})^{-2} , \quad (2.5)$$

where \dot{a} is the time derivative of the scale factor of the universe. The early universe was dominated by radiation for some 70,000 years, and a was proportional to $t^{1/2}$. Afterwards it was dominated by matter for several billion years, with $a \propto t^{2/3}$. Curvature would have become dominant ($\Omega \neq O(1)$) over this time unless

$$|\Omega - 1| \lesssim 10^{-59} \quad (2.6)$$

when the universe began. This is the flatness problem.

The flatness problem is closely related to our original question: without flatness, the universe could not have become large. Suppose, for example, that the early universe had been tuned to flatness less precisely, say $\Omega = 1 + 10^{-20}$. This would have

been a closed universe, which would have expanded to a maximum radius 10^{20} and recollapsed in a big crunch, all within about a time 10^{20} . In other words, this universe would have grown no larger than a proton and lived for less than 10^{-23} seconds.

If the universe had started out slightly underdense (say, $\Omega = 1 - 10^{-20}$), it would have developed a noticeably “open”, i.e., hyperbolic spatial geometry after 10^{-23} seconds, when the largest structures were the size of a proton. After this time, density perturbations would no longer grow and structure formation would cease. The largest coherent structures, each the size of a proton, would freely stream apart. There would be no objects comparable to the size of a planet, let alone galaxies. In this sense, the universe would be small.

A solution to the flatness problem appeared in the early 1980’s: inflation. (It simultaneously addressed a number of other major conundra, such as the horizon problem.) For a detailed treatment, see, e.g., Refs. [8, 9].

The idea is to use Eq. (2.5) to our advantage: if \dot{a} increases with time, then Ω is driven to 1. This can be accomplished by positing that the very early universe was dominated by the vacuum energy of a scalar field before yielding to the standard radiation era. The scale factor grows almost exponentially with time, and Ω quickly approaches 1 with exponential accuracy:

$$|\Omega - 1| \approx e^{-2N} , \quad (2.7)$$

Here N is the number of e -foldings, i.e., e^N is the ratio between the scale factor before and after inflation.

Depending on the energy scale at which inflation occurred, perhaps 60 e -foldings suffice to guarantee that $.1 \leq \Omega \leq 2$ today. But it is easy to write down inflationary models with thousands or millions of e -foldings. In such models, the universe would be spatially flat not only on the present horizon scale, but on exponentially larger scales, which will become visible only after an exponentially longer time than the 13 billion years that have elapsed since the end of inflation.

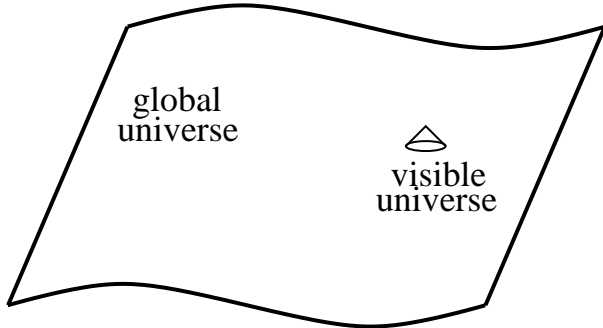


Figure 1: If the early universe underwent inflation, the universe may well be exponentially larger than the visible portion (the interior of our past lightcone).

The true abundance of such models in the potential landscape we get from fundamental theory (Sec. 5) is not yet known. But apparently it is not exceedingly hard to get 60 e -foldings, or else we would have

seen curvature long ago. This suggests that models with more e -foldings are not very rare. It would seem to require some tuning for inflation to have lasted just long enough for the first observable deviations from flatness to occur in the present era. Thus, most inflationary theorists considered $\Omega = 1$ to be a prediction of inflation. By the same token, one would expect that the universe is much larger than the visible universe, perhaps by as much as 10^{100} or 10^{100000} (Fig. 1).

2.2 The cosmological constant problem

When Einstein wrote down the field equation for general relativity,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} \quad (2.8)$$

he had a choice: The cosmological constant Λ was not fixed by the structure of the theory. There was no formal reason to set it to zero, and in fact, Einstein famously tuned it to yield a static cosmological solution—his “greatest blunder”.

The universe has turned out not to be static, and Λ was henceforth assumed to vanish. This was never particularly satisfying even from a classical perspective. The situation is not dissimilar to a famous problem with Newtonian gravity—that there is no formal necessity to equate the gravitational charge with inertial mass.

In any case, the simple fact that the universe is large implies that $|\Lambda|$ is small. I will show this first for the case of positive Λ . Assume, for the sake of argument, that no matter is present ($T_{\mu\nu} = 0$). Then the only isotropic solution to Einstein’s equation is de Sitter space, which exhibits a cosmological horizon of radius

$$R_\Lambda = \sqrt{3/\Lambda} . \quad (2.9)$$

A cosmological horizon is the largest observable distance scale, and the presence of matter will only decrease the horizon radius [10]. We see scales that are large in Planck units, so the cosmological constant must be small in these natural units.

Negative Λ causes the universe to recollapse independently of spatial curvature, on a timescale of order $\Lambda^{-1/2}$. The obvious fact that the universe is old compared to the Planck time then implies that $|\Lambda|$ is small.

These qualitative conclusions do not require any careful measurements. Let us plug in some crude numbers that would have been available already thirty years ago, such as the size of the horizon given in Eq. (2.2), or an age of the universe of order 10^{10} years. They imply that

$$|\Lambda| \lesssim 10^{-122} . \quad (2.10)$$

Hence Λ is very small indeed.

This result makes it tempting to cast scruples aside and simply set $\Lambda = 0$. But from a modern perspective, to eliminate Λ in the classical Einstein equation is not only arbitrary, but futile. Λ returns through the back door, via quantum contributions to the stress tensor, $\langle T_{\mu\nu} \rangle$. It is this effect that makes the cosmological constant problem so notorious.⁴

In quantum field theory, the vacuum is highly nontrivial. Every mode of every field contributes a zero point energy to the energy density of the vacuum (Fig. 2a). The corresponding stress tensor, by Lorentz invariance, must be proportional to the metric:

$$\langle T_{\mu\nu} \rangle = -\rho_\Lambda g_{\mu\nu} . \quad (2.11)$$

Though it appears on the right hand side of Einstein's equation, vacuum energy has the form of a cosmological constant, with $\Lambda = 8\pi\rho_\Lambda$.⁵ Its magnitude will depend on the cutoff.

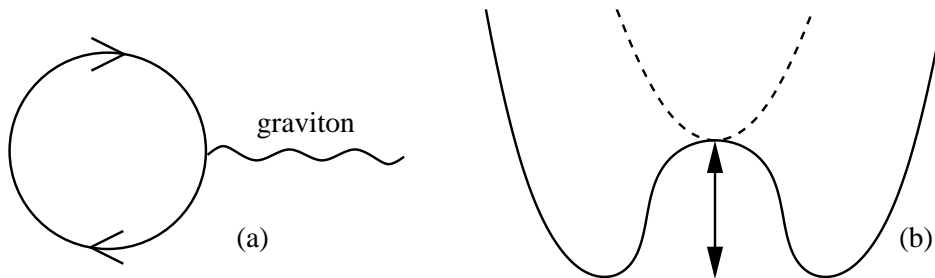


Figure 2: Some contributions to vacuum energy. (a) Virtual particle-antiparticle pairs (loops) gravitate. This is mandated by the equivalence principle, and has been verified experimentally to a high degree of accuracy [13]. The vacuum of the standard model abounds with such pairs and hence should gravitate enormously. (b) Symmetry breaking in the early universe (e.g., the chiral and electroweak symmetry) shifts the vacuum energy by amounts dozens of orders of magnitude larger than the observed value.

For example, consider the electron, which is well understood at least up to energies of order $M = 100$ GeV. Dimensional analysis implies that electron loops up to this cutoff contribute of order $(100 \text{ GeV})^4$ to the vacuum energy, or 10^{-68} in Planck units. Similar contributions are expected from other fields. The real cutoff is probably of order the supersymmetry breaking scale, giving at least a $\text{TeV}^4 \approx 10^{-64}$. It may be as high as the Planck scale, which would yield Λ of order unity. Thus, quantum field theory

⁴In parts, our discussion will follow Refs. [11,12], where more details and references can be found.

⁵This is why the mystery of the smallness of ρ_Λ is usually referred to as the cosmological constant problem. But it would be more appropriate to call it the vacuum energy problem, since the quantum contributions to the vacuum energy are what makes the problem especially hard.

predicts Λ to be some 60 to 120 orders of magnitude larger than the experimental bound, Eq. (2.10).

Additional contributions come from the potentials of scalar fields, such as the potential giving rise to symmetry breaking in the electroweak theory (Fig. 2b). The vacuum energy of the symmetric and the broken phase differ by approximately $(200 \text{ GeV})^4$. Any other symmetry breaking mechanisms at higher or lower energy (such as chiral symmetry breaking of QCD, $(300 \text{ MeV})^4$) will also contribute.⁶

I have exhibited various unrelated contributions to the vacuum energy. Each is dozens of orders of magnitude larger than the empirical bound today, Eq. (2.10). In particular, the radiative correction terms from quantum fields are expected to be at least of order 10^{-64} . They can come with different signs, but it would seem overwhelmingly unlikely for all of them to be carefully arranged to cancel to such exquisite accuracy (10^{-122}) in the present era.

This is the cosmological constant problem: why is the vacuum energy today so small? It represents an immense crisis in physics: a discrepancy between theory and experiment, of 60 to 120 orders of magnitude, in a quantity as basic as the weight of empty space.

2.3 Strategies and predictions

Since the 1980s, various strategies for approaching the cosmological constant problem have been suggested. They fall into two broad classes, with each class facing characteristic challenges and making a characteristic prediction. To give them a fair hearing, let us assume the cosmological data available in the 1980s: the cosmological constant is tightly bounded, but has not yet been measured directly. It might vanish or it might not.

2.3.1 Λ must vanish

The first approach is to seek a universal symmetry principle that requires that $\Lambda = 0$ in our universe today. The problem, of course, is that this challenge has yet to be met. (Supersymmetry guarantees that radiative contributions to the cosmological constant vanish, but in our universe supersymmetry is broken at a scale of at least a TeV.) The

⁶Incidentally, this means that the vacuum energy in the early universe was many orders of magnitude larger than today. This follows from well-tested physics and has been known for a long time, and it should have made us suspicious of the idea that the vacuum energy somehow “had” to be exactly zero. If it was ok to have lots of it a few billion years ago, what could be fundamentally wrong with having some now? It also shows that any mechanism that would set the vacuum energy to zero in the very early universe cannot solve the cosmological constant problem, since $|\Lambda|$ would become huge after symmetry breaking.

challenge is not made easier by the fact that one must allow for a large cosmological constant in the early universe, when various symmetries were not yet broken.

Assuming these challenges could be met, the first approach does make a sharp prediction: $\Lambda = 0$.

2.3.2 Λ is variable

The second strategy [14–16] is to posit that the universe is large—exponentially larger than the presently visible portion—and that Λ varies from place to place, though it can be constant over very large distances. As I will explain below, structure such as galaxies will only form in locations where [16]

$$-10^{-123} \lesssim \rho_\Lambda \lesssim 10^{-121} . \quad (2.12)$$

Since structure is presumably a prerequisite for the existence of observers, we should then not be surprised to find ourselves in such a region.

Why is Λ related to structure formation? To form galaxies and clusters, the tiny density perturbations visible in the cosmic microwave background radiation had to grow under their own gravity, until they became non-linear and decoupled from the cosmological expansion. This growth is logarithmic during radiation domination, and linear in the scale factor during matter domination. Vacuum energy does not get diluted so it inevitably comes to dominate the energy density. As soon as this happens, perturbations cease to grow, and the only structures that remain gravitationally bound are overdense regions that have already gone nonlinear. This means that there would be no structure in the universe if the cosmological constant had been large enough to dominate the energy density before the first galaxies formed [16]. This leads to the upper bound in Eq. (2.12). The lower bound comes about because the universe would have recollapsed into a big crunch too rapidly if the cosmological constant had been large and negative [17].

The problem with the second strategy is twofold:

1. It works only in a theory in which Λ is a dynamical variable whose possible values are sufficiently closely spaced that Eq. (2.12) can be satisfied.
2. Assuming generic initial conditions, one would need to find a mechanism by which at least one value of Λ satisfying Eq. (2.12) can be dynamically attained in a sufficiently large region in the universe.

Supposing that these challenges can be met, one would expect our local cosmological constant to be fairly typical among the possible values of Λ compatible with structure

formation. In an evenly spaced spectrum, most values of Λ satisfying Eq. (2.12) will be of order 10^{-121} ; for example, only a very small fraction will be of order 10^{-146} .

Thus, the “variable Λ ” approach predicts [16] that the cosmological constant is not much smaller than required by Eq. (2.12). This means that it will be large enough to be detectable in the present era. In other words, the “variable Λ ” approach predicts that the vacuum energy should be nonzero and comparable to the matter density today.

3. Precision cosmology

Beginning with the measurement of anisotropies in the cosmic microwave background in the 1990s [18], experimental cosmology has undergone a remarkable transformation. The subject has evolved from order-of-magnitude estimates of a few cosmological parameters to precise measurements of increasingly complex phenomena, leading to the emergence of a “standard model” of cosmology. I will not attempt to review these developments in any detail; see, e.g., Refs. [19–23]. Instead I will summarize how several independent types of observations have helped us evaluate the proposals discussed in the previous section. This is shown schematically in Fig. 3.

3.1 Inflation looks good

1. Measurements of fluctuations in the cosmic microwave background radiation (CMB) strongly support inflation, in two ways:
 - The position of the peaks of the perturbation spectrum as a function of angular scale imply that the universe is spatially flat to excellent precision. Not only is $\Omega \sim O(1)$, but $\Omega = 1$, to accuracy of a few percent. This is the expected result if inflation was the correct explanation of the flatness of the universe.
 - The detailed spectrum of perturbations is nearly scale-invariant and Gaussian. This is natural in inflationary models and rules out many other possible seeds for structure formation, such as topological defects.
2. Measurements of the large scale structure (the distribution of galaxies and galaxy clusters), by techniques such as weak lensing and the Lyman alpha forest, are consistent with $\Omega = 1$ and have reduced the error bars on this result, supporting inflation.
3. Supernova measurements have detected an extra contribution to the total energy density in the universe, $\Omega_\Lambda = .7$. Meanwhile, the observation of large scale struc-

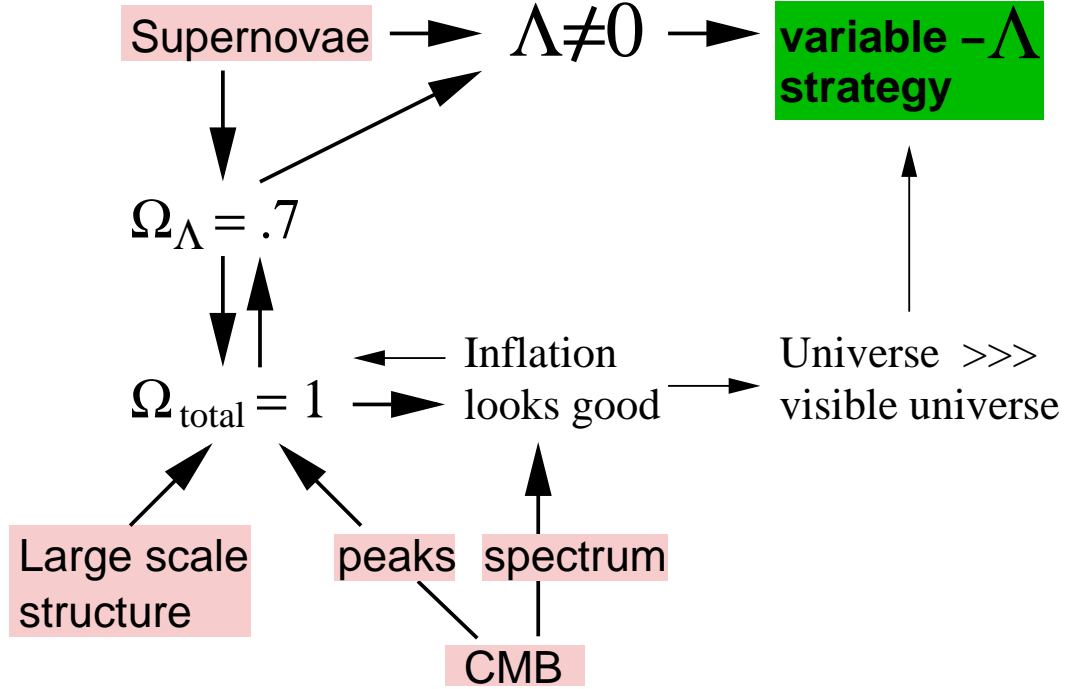


Figure 3: Recent cosmological precision data (light/red shading) strongly support the idea that the cosmological constant is an environmental variable that can scan densely spaced values. The thinner arrows indicate that a result merely adds plausibility to another; the thicker ones denote the most straightforward implication of a result.

ture has corroborated the view that most pressureless matter is dark, $\Omega_{\text{matter}} = .3$. This implies independently of the previous arguments that $\Omega = 1$.

This evidence directly supports inflation. Thus, it indirectly lends credence to the “variable Λ ” approach the cosmological constant problem (Sec. 2.3.2). That strategy requires that the universe be much bigger than what we can presently see of it. As I discussed at the end of Sec. 2.1, this type of global picture is natural in inflationary theory.⁷

3.2 The cosmological constant is non-zero

1. Supernova experiments show that the universe began accelerating its expansion approximately seven billion years ago. This indicates the presence of vacuum energy with $\rho_{\Lambda} = 1.25 \times 10^{-123}$. Present data disfavor any time-dependence

⁷In Sec. 5 I will argue that one should not, in fact, attempt to describe all of this global spacetime at once. Because different regions are forever causally disconnected, they correspond to different outcomes in a decoherent history.

of this component. Thus, the data strongly support the conclusion that the cosmological constant is non-zero.

2. The CMB and large scale structure measurements cited in Sec. 3.1 above reinforce this conclusion, since they imply that $\Omega = 1$. This value cannot be accounted for by dark matter alone. It implies that at least 70% of the universe consists of energy that doesn't clump. The simplest such component is a cosmological constant.
3. Indirectly, this conclusion is also supported by the measurements of the perturbation spectrum in the CMB cited above. They favor inflationary models, and inflation generically predicts $\Omega = 1$.

In summary, there is now strong evidence that $\Lambda > 0$. But a non-zero value of Λ in the observed range is precisely what the “variable Λ ” approach to the cosmological constant problem (Sec. 2.3.2) predicted. This is rather fortunate, since string theory naturally leads to a concrete implementation of the “variable Λ ” strategy, which I will discuss in Sec. 4.

The data essentially rule out the “ Λ -must-vanish” approach (Sec. 2.3.1), since Λ apparently does not vanish. But one could argue that the approach has merely become less appealing, requiring more epicycles to match observation. I will now try to quantify this, before returning to the “variable Λ ” strategy.

3.3 The price of denial

The idea that Λ is an environmental variable is a perfectly logical possibility, but it does represent a retreat. An apt analogy [24] is Kepler's hope of explaining the relation between planetary orbits from first principles. The hope was dashed by Newton's theory of gravitation. Of course, that was no reason to reject a theory of tremendous explanatory power. We simply came to accept that the orbits are the results of historical accidents and that there are many other solar systems in which different possibilities are realized.

But let us not be too hasty in abandoning the quest for a unique prediction of today's value of Λ . Instead, let us ask what it would take to maintain this type of approach in light of the discovery of non-zero vacuum energy.

We would need to assume that some symmetry or other effect makes Λ vanish, except for a correction of order 10^{-123} . This takes a miracle as the starting point: despite decades of work, no mechanism has been found that requires $\Lambda = 0$ without running into conflict with known physics [11, 13]. And supposing it existed, how would any posited correction evade a mechanism so powerful as to cancel out many enormous

and disparate contributions to vacuum energy (see Sec. 2.2)? Finally, why does this correction have just the right magnitude so as to be comparable to the matter density at the present time?

In short, the price of insisting on a unique prediction for the cosmological constant is that the cosmological constant problem breaks up into three problems, none of them solved:⁸

1. What makes the cosmological constant vanish?
2. Why is the cosmological constant not exactly zero?
3. Why now?

The first of these three problems seems by far the hardest; in any case, it has resisted several decades of attack. It is tempting to assume it solved, and to speculate instead about the putative correction that makes Λ nonzero. But let us be mindful that any results obtained in this manner will rest on wishful thinking.

Among such approaches, dynamical scalar fields (“quintessence”) take a prominent role, perhaps because they posit observable deviations from the equation of state of a cosmological constant. I confess that I find this development perplexing. Dynamical scalars do not match the data better than a fixed cosmological constant, and they are theoretically far more baroque.

Scalar fields like to roll off to infinity rapidly, or quickly get stuck in a local minimum. For a scalar to mimic vacuum energy and yet exhibit nontrivial dynamics more than ten billion years after the big bang, would require an extremely flat (but not exactly flat) potential over an enormous range. This necessitates tunings [25–27] that include, but go far beyond, arbitrarily setting the present vacuum energy to a small value. Yet further tuning [26] is needed to explain why the long-range force associated with an almost massless scalar has not been detected.⁹

⁸In some discussions, the cosmological constant problem is identified with these three questions. But this implicitly assumes that Λ is unique. Fundamentally, the cosmological constant problem is only one question: why is the vacuum energy not huge? As I explained in Sec. 2.3.2, the “variable Λ ” approach predicts that Λ will be small, but large enough to be already noticeable in our era. Thus it avoids the first in our list of questions; it answers the third before we have a chance to worry about it; and the second question does not arise. Indeed, at present it is senseless to ask why $\Lambda \neq 0$, since we know of no reason why Λ *should* vanish. That it is asked anyway betrays only how deeply we had absorbed the prejudice that it does.

⁹Some authors do confront these latter problems (see Refs. [28, 29] for recent examples). Aside from the unsolved theoretical question of why Λ should vanish at late times, such models also receive increasing pressure from observation, since dark energy does appear to be at least approximately constant.

Thus, quintessence not only fails to address the very real question of why Λ is small, but, unprovoked by data, burdens us with the challenge of explaining several additional very small numbers.

Understandably, experimenters demand parametrizations of some spaces of models that they can hope to constrain [30]. But let us not confuse models (which come cheaper the more complicated we make them) with explanations. The danger is that we will forever abuse the data to constrain ever more baroque models while overlooking the simplest one [31].

A cosmological constant is already favored by experiment, and it is arguably the only model for which we have at least a tentative fundamental explanation (Sec. 4). If one finds this explanation unattractive, it makes sense to seek a different origin of the simplest model compatible with the data. What makes no sense is to write down more complicated models than the data require, while making no attempt to explain their origin in a credible fundamental theory.¹⁰

I am not, of course, proposing that we stop looking experimentally for any time dependence of dark energy. The evidence for a nonzero cosmological constant is surely among the most profound insights ever gained from experiment. This alone warrants every effort to confirm and refine what we know about dark energy. Perhaps more surprises await us, complicating the story further. Meanwhile, I feel that we theorists would do well to solve the problems we actually have; those are bad enough.

4. The discretuum

I have argued that experiment favors the “variable Λ ” approach to the cosmological constant problem. I have also spelled out the main challenges to its implementation. In this section, I will present evidence that these challenges are met by string theory. Large parts of this section are based on joint work with J. Polchinski [3].

4.1 A continuous spectrum of Λ ?

The first task is to show that the cosmological constant can take on a sufficiently dense “discretuum” of values. In string theory, each line in the spectrum of Λ will correspond to a long-lived metastable vacuum.

¹⁰Similar remarks apply to the idea that gravity should be modified to account for the apparent deviations from $\Lambda = 0$. This approach also makes sense only to the degree that we have any reason to believe that Λ should vanish at late times, which we don’t. In a modified gravity theory, the quantum field theory contributions to the cosmological constant would be just as large, unless one violates the equivalence principle, which conflicts with other experiments [13].

Why look for a discretuum and not a continuum of values? The quick answer is that we can plausibly realize a discretuum in string theory, but not a continuum. In fact, we know of no adjustable parameters on which the cosmological constant depends in a continuous manner—at least if our goal at the same time is metastability [3].

But why insist on metastability? I will give a brief argument that we have good reasons to do so. This shows more generally that it would be difficult to realize the “variable Λ ” approach with a continuous spectrum.

If the continuous parameter is like an integration constant, fixed once and for all, then it will not allow Λ to vary between large regions in the universe, so it would have to be tuned by hand. If the parameter can change over time, then the vacuum energy can be lowered by sliding down the spectrum continuously. But this is tantamount to introducing a scalar field potential, and it leads to versions of problems described in Sec. 3.3: Why, in ten billion years, has Λ not relaxed to its lowest possible value? (We cannot assume that this “ground state” is the observed value, or zero, since this would beg the question; radiative corrections would immediately destroy such a setup.)

It is difficult to see how such a special behavior could be arranged, other than in a theory with many metastable vacua, but this would get us back to the discrete case. Moreover, even with anthropic constraints there is no reason why Λ should change as slowly as current bounds indicate. Thus, one would predict a universe with blatantly time-dependent vacuum energy. In the discretuum, on the other hand, the minimum value of the cosmological constant naturally remains fixed for the lifetime of the metastable vacuum, which can easily exceed ten billion years.

4.2 A single four-form field

To begin, I will present a very simple model of a discretuum. This model will not work for two reasons: it cannot be realized in string theory, and it produces an empty universe [32, 33]. Nevertheless, it will be instructive, and it invites a useful analogy with electromagnetism.

Recall that the Maxwell field, F_{ab} , is derived from a potential, $F_{ab} = \partial_a A_b - \partial_b A_a$. The potential is sourced by a point particle through a term $\int e \mathbf{A}$ in the action, where the integral is over the worldline of the particle, and e is the charge. Technically, \mathbf{F} is a two-form (a totally antisymmetric tensor of rank 2), and \mathbf{A} is a one-form coupling to a one-dimensional worldvolume (the worldline of the electron).

The field content of string theory and supergravity is completely determined by the structure of the theory. It includes a four-form field, F_{abcd} , which derives from a three-form potential:

$$F_{abcd} = \partial_{[a} A_{bcd]} , \quad (4.1)$$

where square brackets denote total antisymmetrization. This potential naturally couples to a two-dimensional object, a membrane, through a term $\int q \mathbf{A}$, where the integral runs over the 2+1 dimensional membrane worldvolume, and q is the membrane charge.

The properties of the four-form field in our 3+1 dimensional world mirror the behavior of Maxwell theory in a 1+1 dimensional system. Consider, for example, an electric field between two capacitor plates. Its field strength is constant both in space and time. Its magnitude depends on how many electrons the negative plate contains; thus it will be an integer multiple of the electron charge: $E = ne$.

Its energy density will be one half of the field strength squared:

$$\rho = \frac{F_{ab}F^{ab}}{2} = \frac{n^2 e^2}{2} \quad (4.2)$$

In order to treat this as a system with only one spatial dimension, I have integrated over the directions transverse to the field lines, so ρ is energy per unit length. The pressure is equal to $-\rho$. The corresponding 1+1 dimensional stress tensor has the form of Eq. (2.11), so the electromagnetic stress tensor acts like vacuum energy in 1+1 dimensions.

The same is true for the four-form in our 3+1 dimensional world. First of all, the equation of motion in the absence of sources is $\partial_a(\sqrt{-g}F^{abcd}) = 0$, with solution

$$F^{abcd} = c\epsilon^{abcd}, \quad (4.3)$$

where ϵ is the unit totally anti-symmetric tensor and c is an arbitrary constant. In string theory, there are “magnetic” charges (technically, five-branes) dual to the “electric charges” (the membranes) sourcing the four-form field. Then, by an analogue of Dirac quantization of the electric charge, one can show that c is quantized in integer multiples of the membrane charge, q :

$$c = nq. \quad (4.4)$$

Note that the actual value of the four-form field is thus quantized, not only the difference between possible values.

The four-form field strength squares to $F_{abcd}F^{abcd} = 24c^2$, and the stress tensor is proportional to the metric, with

$$\rho = \frac{1}{2 \times 4!} F_{abcd}F^{abcd} = \frac{n^2 q^2}{2} \quad (4.5)$$

In summary, the four-form field is non-dynamical, and it contributes $n^2 q^2/2$ to the vacuum energy. It is thus indistinguishable from a contribution to the cosmological constant.

Next, let us include non-perturbative quantum effects. The electric field between the plates will be slowly discharged by Schwinger pair creation of field sources. This is a process by which a electron and a positron tunnel out of the vacuum. Since field lines from the plates can now end on these particles, the electric field between the two particles will be lower by one unit [$ne \rightarrow (n-1)e$]. The particles appear at a separation such that the corresponding decrease in field energy compensates for their combined rest mass. They are then subjected to constant acceleration by the electric field until they hit the plates. If the plates are far away, they will move practically at the speed of light by that time.

For weak fields, this tunneling process is immensely suppressed, with a rate of order $\exp(-\pi m^2/ne^2)$, where the exponent arises as the action of a Euclidean-time solution describing the appearance of the particles. Thus, a long time passes between creation events. However, over large enough time scales, the electric field will decrease by discrete steps of size e . Correspondingly, the 1+1 dimensional “vacuum energy”, i.e., the energy per unit length in the electric field, will decrease by a discrete amount $[n^2e^2 - (n-1)^2e^2]/2 = (n - \frac{1}{2})e^2$. Note that this step size depends on the remaining flux.

Precisely analogous nonperturbative effects occur for the four-form field in 3+1 dimensions. By an analogue of the Schwinger process, spherical membranes can spontaneously appear. (This is the correct analogue: the two particles above form a zero-sphere, i.e., two points; the membrane forms a two-sphere.) Inside this source, the four-form field strength will be lower by one unit of the membrane charge [$nq \rightarrow (n-1)q$]. The process conserves energy: the initial membrane size is such that the membrane mass is balanced against the decreased energy of the four-form field inside the membrane. The membrane quickly grows to convert more space to the lower energy density, expanding asymptotically at the speed of light.

Membrane creation is a well-understood process described by a Euclidean instanton, and like Schwinger pair creation, is generically exponentially slow. Ultimately, however, it will lead to the step-by-step decay of the four-form field. Inside a new membrane, the vacuum energy will be lower by $(n - \frac{1}{2})q^2$.

This suggests a mechanism for cancelling off the cosmological constant. Let us collect all contributions (see Sec. 2.2), except for the four-form field, in a “bare” cosmological constant λ . Generically, $|\lambda|$ should be of order unity (at least in the absence of supersymmetry), and we will assume without excessive loss of generality that it is negative. With n units of four-form flux turned on, the full cosmological constant will be given by

$$\Lambda = \lambda + \frac{1}{2}n^2q^2 \tag{4.6}$$

If n starts out large, the cosmological constant will decay by repeated membrane creation, until it is close to zero. The smallest value of $|\Lambda|$ is attained for the flux n_{best} , given by the nearest integer to $\sqrt{2|\lambda|}/q$. The step size near $\Lambda = 0$ is thus given by $(n_{\text{best}} - \frac{1}{2})q^2$. For this mechanism to produce a value in the Weinberg window, Eq. (2.12), this step size would need to be of order 10^{-121} or smaller. This requires an extremely small membrane charge,

$$q < 10^{-121} |\lambda|^{-1/2} \quad (4.7)$$

(the bare cosmological constant λ is at best of order one).

This leads to two problems [32, 33]: the small-charge problem, and the empty-universe problem. The membrane charge q is now itself exceedingly small and thus unnatural. In particular, despite attempts in this direction [34], it is not known how to realize such a small charge in string theory.

Assuming the small-charge problem could be resolved, the mechanism would lead to a universe very different from ours: it would be devoid of all matter and radiation. The point is that small values of Λ are approached very gradually from above. Thus the universe is dominated by positive vacuum energy all along, leading to accelerated expansion. The exponential suppression of membrane nucleation events ensures that this expansion goes on long enough to dilute all matter. Eventual membrane nucleation decreases the vacuum energy only by a tiny amount (10^{-121} or less). At best, this might reheat the universe to 10^{-30} , or about 10^{-2} eV.¹¹ This falls well short of the 10 MeV mark necessary to make contact with standard cosmology, a theory we trust at least back to nucleosynthesis.

4.3 Multiple four-form fields

The above problems can be overcome by considering a theory with more than one species of four-form field. I will explain why this situation arises naturally in string theory, but first I will discuss how multiple four-form fields can produce a dense discretuum without requiring small charges.

Consider a theory with J four-form fields. Correspondingly there will be J types of membrane, with charges q_1, \dots, q_J . Above I analyzed the case of a single four-form field; essentially the conclusions still apply to each field separately. In particular, each field strength separately will be constant in 3+1 dimensions,

$$F_{(i)}^{abcd} = n_i q_i \epsilon^{abcd} \ , \quad (4.8)$$

¹¹The actual number is vastly smaller still, since most of the energy goes into accelerating the growth of the membrane bubble. This is the reason why the empty-universe problem also plagues “old inflation” [35], even though the jump in vacuum energy is considerably larger in that case.

and it will contribute like vacuum energy to the stress tensor.

Let us again collect all contributions to vacuum energy, *except* for those from the J four-form fields, in a bare cosmological constant λ , which I assume to be negative but otherwise generic (i.e., of order unity). Then the total cosmological constant will be given by

$$\Lambda = \lambda + \frac{1}{2} \sum_{i=1}^J n_i^2 q_i^2 . \quad (4.9)$$

This will include a value in the Weinberg window, Eq. (2.12), if there exists a set of integers n_i such that

$$2|\lambda| < \sum n_i^2 q_i^2 < 2(|\lambda| + \Delta\Lambda) , \quad (4.10)$$

where $\Delta\Lambda \approx 10^{-121}$.

A nice way to visualize this problem is to consider a J -dimensional grid, with axes corresponding to the field strengths $n_i q_i$, as shown in Fig. 4. Every possible

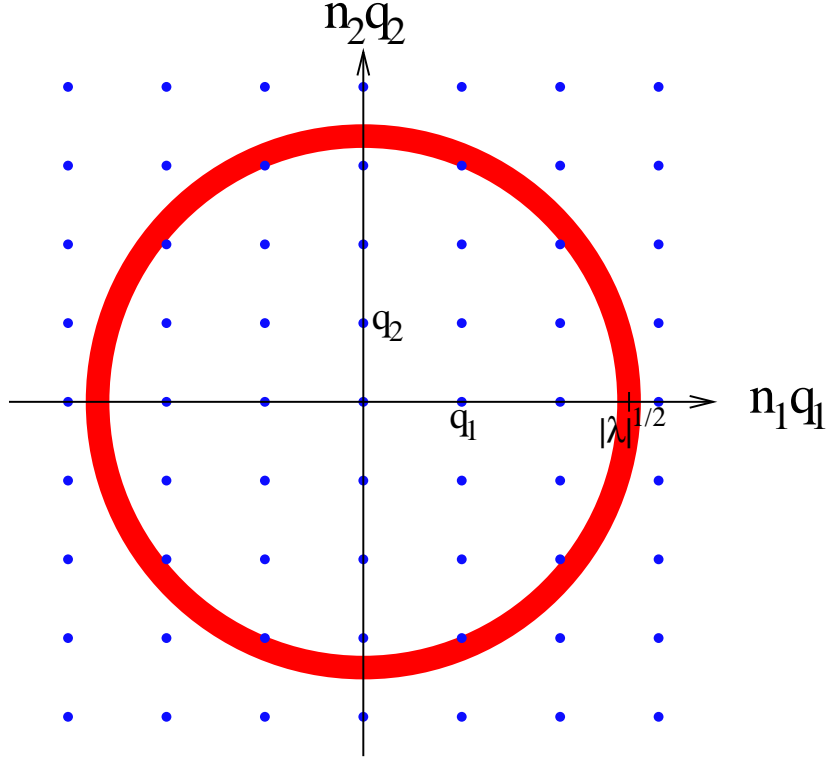


Figure 4: Possible configurations of the four-form fluxes correspond to discrete points in a J -dimensional grid. By Eq. (4.9), vacua that allow for structure formation lie within a thin shell of radius $\sqrt{2|\lambda|}$ and width $\Delta\Lambda/\sqrt{2|\lambda|}$, where λ is the bare cosmological constant and $\Delta\Lambda$ is the width of the Weinberg window, Eq. (2.12).

configuration of the four-form fields corresponds to a list of integers n_i , and thus to a discrete grid point. The Weinberg window can be represented as a thin shell of radius $\sqrt{2|\lambda|}$ and width $\Delta\Lambda/\sqrt{2|\lambda|}$. The shell has volume

$$V_{\text{shell}} = \Omega_{J-1} (\sqrt{2|\lambda|})^{J-1} \frac{\Delta\Lambda}{\sqrt{2|\lambda|}} = \Omega_{J-1} |2\lambda|^{\frac{J}{2}-1} \Delta\Lambda , \quad (4.11)$$

where $\Omega_{J-1} = 2\pi^{J/2}/\Gamma(J/2)$ is the area of a unit $J-1$ dimensional sphere. The volume of a grid cell is

$$V_{\text{cell}} = \prod_{i=1}^J q_i . \quad (4.12)$$

There will be at least one value of Λ in the Weinberg window, if $V_{\text{cell}} < V_{\text{shell}}$, i.e., if

$$\frac{\prod_{i=1}^J q_i}{\Omega_{J-1} |2\lambda|^{\frac{J}{2}-1}} < |\Delta\Lambda| . \quad (4.13)$$

The most important consequence of this formula is that charges no longer need to be very small. I will shortly argue that in string theory one naturally expects J to be in the hundreds. With $J = 100$, for example, Eq. (4.13) can be satisfied with charges q_i of order $10^{-1.6}$, or $\sqrt{q_i} \approx 1/6$ (the latter has mass dimension 1 and so seems an appropriate variable for the judging naturalness of this scenario). Interestingly, the large expected value of the bare cosmological constant is actually welcome: it becomes more difficult to satisfy Eq. (4.13) if $|\lambda| \ll 1$.

The origin of the large number of four-form fields lies in the topological complexity of small extra dimensions. String theory is most naturally formulated in 9+1 or 10+1 spacetime dimensions. For definiteness I will work with the latter formulation (also known as M-theory). If it describes our world, then 7 of the spatial dimensions must be compactified on a scale that would have eluded our most careful experiments. Thus one can write the spacetime manifold as a direct product:

$$M = M_{3+1} \times X_7 . \quad (4.14)$$

Typically, the compact seven-dimensional manifold X_7 will have considerable topological complexity, in the sense of having large numbers of non-contractable cycles of various dimensions.

To see what this will mean for the 3+1 dimensional description, consider a string wrapped around a one-cycle (a “handle”) in the extra dimensions. To a macroscopic observer this will appear as a point particle, since the handle cannot be resolved. Now, recall that M-theory contains five-branes, the magnetic charges dual to membranes.

Like strings on a handle, five-branes can wrap higher-dimensional cycles within the compact extra dimensions. A five-brane wrapping a three-cycle (a kind of non-contractible three-sphere embedded in the compact manifold) will appear as a two-brane, i.e., a membrane, to the macroscopic observer.

Six-dimensional manifolds, such as Calabi-Yau geometries, generically have hundreds of different three-cycles, and adding another dimension will only increase this number. The five-brane—one of a small number of fundamental objects of the theory—can wrap any of these cycles, giving rise to hundreds of apparently different membrane species in 3+1 dimensions, and thus, to $J \sim O(100)$ four-form fields, as required.

The charge q_i is determined by the five-brane charge (which is set by the theory to be of order unity), the volume of X_7 , and the volume of the i -th three-cycle. The latter factors can lead to charges that are slightly smaller than 1, which is all that is required. Note also that the volumes of the three-cycles will generically differ from each other, so one would expect the q_i to be mutually incommensurate. This is important to avoid huge degeneracies in Eq. (4.9).

Each of the flux configurations (n_1, \dots, n_J) corresponds to a metastable vacuum. Fluxes can only change if a membrane is spontaneously created. As discussed in Sec. 4.2, this Schwinger-like process is generically exponentially suppressed, leading to extremely long lifetimes. Thus, multiple four-forms naturally give a dense discretuum of metastable vacua.

The model I have presented is an oversimplification. When it was first proposed, it was not yet understood how to stabilize the compact manifold against deformations (technically, how to fix all moduli fields including the dilaton). This is clearly necessary in any case if string theory is to describe our world. But one would expect that in a realistic compactification, the fluxes wrapped on cycles should deform the compact manifold, much like a rubber band wrapping a doughnut-shaped balloon. Yet, I have pretended that X_7 stays exactly the same independently of the fluxes n_i .

Therefore, Eq. (4.9) will not be correct in a more realistic model. The charges q_i , and indeed the bare cosmological constant $|\lambda|$, will themselves depend on the integers n_i . Thus the cosmological constant may vary quite unpredictably. But the crucial point remains unchanged: the number of vacua, N , can be extremely large, and the discretuum should have a typical spacing $\Delta\Lambda \approx 1/N$. For example, if there are 500 three-cycles and each can support up to 9 units of flux, there will be of order $N = 10^{500}$ metastable configurations. If their vacuum energy is effectively a random variable with at most the Planck value ($|\Lambda| \lesssim 1$), then there will be 10^{380} vacua in the Weinberg window, Eq. (2.12).

In the meantime, there has been significant progress with stabilizing the compact geometry (e.g., Refs. [36, 37]; see Refs. [6, 38, 39] for reviews.). In particular, Kachru,

Kallos, Linde, and Trivedi [4] have shown that metastable de Sitter vacua can be realized in string theory while fixing all moduli.¹² Their construction supports the above argument that the number of flux vacua can be extremely large. More sophisticated counting methods [42] bear out the quantitative estimates obtained from the simple model I have presented.

I will close with two remarks. The need for extra dimensions could be regarded as an unpleasant aspect of string theory, since it forces us to worry about why and how they are hidden. Ironically, they are precisely what has allowed string theory to address the cosmological constant problem and pass its first observational test.

One often hears that there are now 10^{500} “string theories”, suggesting a loss of fundamental simplicity and uniqueness. This is like saying that there are myriads of standard models because there are many ways to make a lump of iron. From five standard model particles, one can construct countless metastable configurations of atoms, molecules, and condensed matter objects. Similarly, the large number of vacua in string theory arises by combining a small set of fundamental ingredients in different ways, *in the extra dimensions*. From this perspective, numbers like 10^{500} should not surprise us.

4.4 Our way home

I have argued that string theory contains such a dense spectrum of metastable vacua that many of them will satisfy the Weinberg inequality, Eq. (2.12). But still, they represent only a very small fraction of the total number of vacua. Hence, there is no particular reason to assume that the universe would have started out in one of the relatively rare vacua with small late-time cosmological constant. Such an assumption would be especially problematic since the late-time value of the cosmological constant is initially far from apparent. In our own vacuum, for example, the cosmological constant is now small but was enormously larger at early times, before inflation ended and various symmetries were broken.

Fortunately, it is unnecessary to assume that the universe starts out in a Weinberg vacuum. I will now show that starting from generic initial conditions, the universe will grow arbitrarily large. Over time, it will come to contain enormous regions (“bubbles” or “pockets”) corresponding to each metastable vacuum (Fig. 5). In particular, the Weinberg vacua will be realized somewhere in this “multiverse”. It will be seen that these vacua can be efficiently reheated, so the empty-universe problem of Sec. 4.2 will not arise.

By Eq. (4.9), all but a finite number of metastable vacua will have $\Lambda > 0$. Let us assume that the universe begins in one of these vacua. Of course, this means that

¹²Constructions in non-critical string theory (i.e., string theory with more than ten spacetime dimensions) were proposed earlier [40, 41].

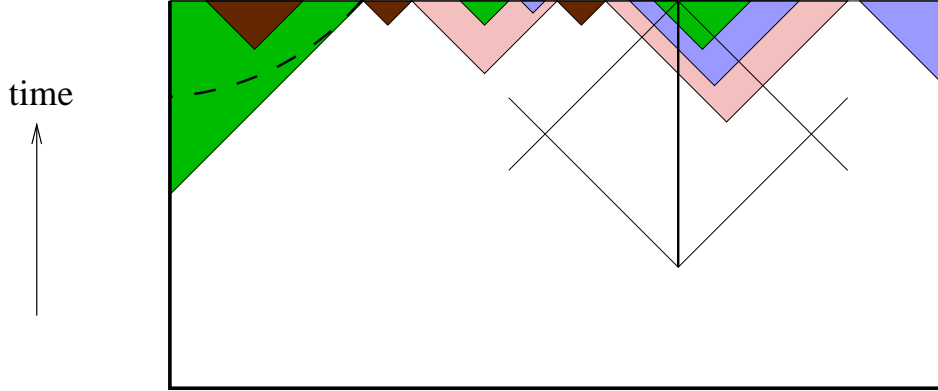


Figure 5: Bird’s eye view of the universe. There are regions corresponding to every vacuum in the landscape (shown in different colors). Each region is an infinite, spatially open universe; the dashed line shows an example of an instant of time. The black diamond is an example of a spacetime region that is causally accessible to a single observer (see Sec. 5).

typically the cosmological constant will be large initially. Since $\Lambda > 0$, the universe will be well described by de Sitter space. It can be thought of as a homogeneous, isotropic universe expanding exponentially on a characteristic time scale $\Lambda^{-1/2}$.

Every once in a long while (this time scale being set by the action of a membrane instanton, and thus typically much larger than $\Lambda^{-1/2}$), a membrane will spontaneously appear and the cosmological constant will jump by $(n_i - \frac{1}{2})q_i^2$. But this does not affect the whole universe. Λ will have changed only inside the membrane bubble. This region grows arbitrarily large as the membrane expands at the speed of light.

But crucially, this does *not* imply that the whole universe is converted into the new vacuum [43]. This technical result can be understood intuitively. The ambient, old vacuum is still, in a sense, expanding exponentially fast. The new bubble eats up the old vacuum as fast as possible, at nearly the speed of light. But this is not fast enough to compete with the background expansion.

More and more membranes, of up to J different types, will nucleate in different places in the rapidly expanding old vacuum. Yet, there will always be some of the old vacuum left. One can show that the bubbles do not “percolate”, i.e., they will never eat up all of space [35]. Thus different fluxes can change, and different directions in the J -dimensional flux space are explored.

Inside the new bubbles, the game continues. As long as Λ is still positive, there is room for everyone, because the background expands exponentially fast. In this way, all the points in the flux grid (n_1, \dots, n_J) , are realized as actual regions in physical space. The cascade comes to an end wherever a bubble is formed with $\Lambda < 0$, but this affects

only the interior of that particular bubble (it will undergo a big crunch). Globally, the cascade continues endlessly.

Perhaps surprisingly, each bubble interior is an open FRW universe in its own right, and thus infinite in spatial extent.¹³ Yet, each bubble is embedded in a bigger universe (sometimes called “multiverse” or “megaverse”), which is extremely inhomogeneous on the largest scales.

An important difference to the model with only one four-form is that the vacua will not be populated in the order of their vacuum energy. Two neighboring vacua in flux space (i.e., neighbors in the “landscape”), will differ hugely in cosmological constant. That is, they differ by one unit of flux, and the charges q_i are not much smaller than one, so by Eq. (4.9) this translates into an enormous difference in cosmological constant. Conversely, vacua with very similar values of the cosmological constant will be well separated in the flux grid (i.e., far apart in the landscape).

This feature is crucial for solving the empty universe problem. When our vacuum was produced in the interior of a new membrane, the cosmological constant may have decreased by as much as 1/100 of the Planck density. Hence, the temperature before the jump was enormous (in this example, the Gibbons-Hawking temperature of the corresponding de Sitter universe would have been of order 1/10 of the Planck temperature), and only extremely massive fields will have relaxed to their minima. Most fields will be thermally distributed and can only begin to approach equilibrium after the jump decreases the vacuum energy to near zero.

Thus, the final jump takes on the role analogous to the big bang in standard cosmology. The “universe” (really, just our particular bubble) starts out hot and dense. If the effective theory in the bubble contains scalar fields with suitable potentials, there will be a period of slow-roll inflation as their vacuum energy slowly relaxes. (This was apparently the case in our vacuum.) At the end of this slow-roll inflation process, the universe reheats.

To a (purely hypothetical) observer in the primordial era of a given bubble, it would be far from obvious what the late-time cosmological constant will be, since this depends on future symmetry breakings and the relaxation of scalar field potential energy. The small late-time values in some bubbles are the result of purely accidental cancellations—which are bound to happen in some vacua if there are 10^{500} vacua in total.

¹³In an open universe, spatial hypersurfaces of constant energy density are three-dimensional hyperboloids. This shape is dictated by the symmetries of the instanton describing the membrane nucleation. It is closely related to the hyperbolic shape of the spacetime paths of accelerating particles, like the electron-positron pair studied above.

To a hypothetical primordial observer in our own bubble, the evolution of vacuum energy would seem like a sequence of bizzare coincidences. I assume here that the observer is sufficiently intelligent to know that quantum field theory predicts a cosmological constant of order one. In the primordial era, the energy density in radiation is large, and it could mask even a fairly large cosmological constant. But as the universe cools off, a cosmological constant exceeding the ever decreasing energy density in matter and radiation would become immediately apparent. Thus, the discrepancy between theory and observation grows larger and larger.

Much to his surprise, our observer would find the vacuum energy in the minimum of the inflationary potential to be much smaller than during inflation—in fact, it cannot be distinguished from zero. (This allows the universe to reheat, without immediately inflating all matter away, but why would our observer care?) During electroweak symmetry breaking, at time 10^{-12} sec, the vacuum energy density shifts by $(200 \text{ GeV})^4$. Our observer computes this and is thus led to expect that soon afterwards, when the radiation energy drops below $(200 \text{ GeV})^4$, the dynamical effects of a cosmological constant will finally become apparent. It does not, so the observer is forced to conclude that the shift must have cancelled against another, equally large contribution that he had not noticed earlier since radiation was too dense. In fact, the cancellation is so exquisite that vacuum energy remains dynamically irrelevant at the much later time 1 sec. (This allows nucleosynthesis to proceed.) After hundreds of millions of years, at vastly lower energy density, still no vacuum energy is apparent (allowing for the formation of galaxies to proceed undisturbed). Only after billions of years (after structure has formed), does vacuum energy resurface and begin to dominate over the ever more dilute matter energy density.

If such hypothetical observers existed, this sequence really *would* be bizzare and unexpected. There are far more vacua with similar primordial evolution but without the anomalously small late-time cosmological constant. All the corresponding bubbles would presumably harbor similar primordial observers. Then the vast majority of observers would *not* see a sequence of “miracles” leading to a late-time cosmological constant as small as 10^{-121} .

But it appears that no such hypothetical primordial observers exist. Observers will arise only after some structure has formed. This happens only in the “bizzare”, rare vacua in which accidental cancellations produce a late-time cosmological constant of order 10^{-121} or less. Any larger, and vacuum energy would disrupt galaxy formation. We should not be surprised, therefore, to find ourselves in such a bubble.

5. The landscape and predictivity

5.1 A new challenge

A good explanation will do more than solve a problem. It should offer us a new way of thinking, and in doing so, raise new, interesting problems. In fact, the picture I have outlined does present a tremendous challenge: how does one make predictions in the landscape?

Let us suppose that there are 10^{500} metastable vacua. Among them, everything varies: forces, coupling strengths, masses, field content, gauge groups, and other aspects of the low energy Lagrangian. Are the “constants” of nature we measure constrained by nothing but the fact of our existence? This would be a bleak prospect indeed.

In order to look at the problem dispassionately, it helps to take recourse once more to the analogy with complex, many-particle systems developed near the end of the previous section. A vast number of phenomena arise from a few particles in the standard model: the world is a rich, complex place. But this does not imply that anything goes. There are only a finite number of elements, and a random combination of atoms is unlikely to form a stable molecule. Even quantities such as material properties ultimately derive from standard model parameters and cannot be arbitrarily dialed.

Similarly, one would expect that there are low-energy Lagrangians that simply cannot arise from string theory with its limited set of ingredients, no matter how complicated the manner in which they are combined [44–46].

Moreover, the great complexity of a system need not be an obstacle to its effective description. Imagine we had never heard of thermodynamics and were told to describe the behavior of all the air molecules in a room. Or suppose we were ignorant of condensed matter physics, and were charged with deriving the properties of metals from the standard model. Would we not worry, for a moment, that these tasks are too complex to be tractable? Of course, we know well that such problems yield to the laws of large numbers. The predictive power of statistical or effective theories is completely deterministic in practice: not in ten billion years will the air ever collect in one corner of the room. This is not to say that finding such descriptions is trivial, only that it is possible.

Similarly, there is every reason to hope that a set of 10^{500} vacua will yield to statistical reasoning, allowing us to extract predictions. Yet we must not presume this task simple or even straightforward. We are just beginning, so the present scarcity of predictions is hardly proof of their impossibility.

The problem can be divided into three separate tasks:

1. Statistical properties of the string theory landscape

2. Selection effects from cosmological dynamics

3. Anthropic selection effects

The first of these has been tackled by a number of authors; see, e.g., Refs. [42, 47–49], or Ref. [50] for a review. The question is, what is the relative abundance of stable or metastable vacua with specified low-energy properties. Our understanding of metastable vacua is still rather qualitative, so most investigations focus on supersymmetric vacua instead, which are under far better control. Clearly, it would be desirable to extend our samples; this will likely require significant progress in understanding vacua without supersymmetry. Meanwhile it will be interesting to understand the extent to which current samples are representative of more realistic vacua, especially since one is usually working in a particular corner of moduli space.

This remains a very active area of research, and I will not attempt a more detailed review. Next, I will discuss a recent approach to the second and third task.

5.2 Probabilities in eternal inflation

It is not enough to calculate the probability that a random metastable vacuum picked from the theory landscape has a given property. Cosmological dynamics is interposed between the theory landscape and the actual realization of vacua as large regions in the universe. This dynamical process may preferentially produce some vacua and suppress others. This is the second question listed above: What is the relative abundance of different vacua *in the physical universe*?

Computational difficulties aside, this question turns out to be hard to answer even in principle, because of a scourge of infinities. The global structure of the universe arising from the string landscape is extremely complicated (see Sec. 4.4). Each vacuum i is realized infinitely many times as a bubble embedded in the global spacetime. Moreover, every bubble is an open universe and thus of infinite spatial extent.

The most straightforward way of regulating the infinities is to consider the universe at finite time before taking a limit. There is an ambiguity in whether one should compare the volumes, or simply the number of each type of bubble on this time slice (or some intermediate quantity). Worse, results depend strongly on the choice of time variable [51, 52], and no preferred time-slicing is available in the highly inhomogeneous global spacetime.

A number of slicing-invariant probability measures have been proposed; see, e.g. [53–55] for recent work. Yet, slicing invariance is far from a strong enough criterion for determining a unique measure; for example, any function of an invariant measure will again be invariant.

In addition to these severe ambiguities, known slicing-invariant proposals appear to lead to predictions that disagree with observation [56–59]. The first problem arises in proposals where the probability carried by a vacuum is proportional to the factor by which inflation increases the volume. (This refers to the ordinary slow-roll inflation of Sec. 2.1, not the false-vacuum driven eternal inflation of Sec. 4.4.) This factor is exponential in the duration of inflation. In Ref. [56] it was argued that generically, both the number of e -foldings and the density perturbations produced will depend monotonically on parameters of the inflationary model. Thus, the great weight carried by long periods of inflation should push the density contrast $\delta\rho/\rho$ towards 0 or 1. One can argue that life would be impossible in a universe with $\delta\rho/\rho$ too small or too large [60]. But anthropic arguments cannot resolve the paradox. The exponential preference for extreme values means that we should live dangerously, a lucky fluctuation in an inhospitable universe. Instead, the density contrast in our universe appears to be comfortably within the anthropic window.

A more severe problem arises, e.g., in the proposal by Garriga et al. [54]: One can show that the overwhelming majority of observers are not like us but arise from random fluctuations [58]. Assuming that we are typical observers (as we must if we want to make any predictions), this conflicts with observation. It could be avoided if all vacua that can harbor observers decay on a timescale not much longer than $\Lambda^{-1/2}$. But this is extremely implausible in the string landscape [59].

Recently, a local (or “causal”, or “holographic”) approach has been developed which avoids the ambiguities and resolves the paradoxes described above [59, 61–63]. Its original motivation, however, comes from the study of black hole evaporation, which appears to be a unitary process [64, 65]. A different kind of paradox arose in this context: The initial quantum state is duplicated, appearing at the same instant of time both in the Hawking radiation and inside the black hole. However, causality prevents any observer from seeing both copies. Thus, the black hole paradox is resolved if we give up on trying to describe the spacetime globally [66, 67]. Indeed, all that is needed is a theory that can describe the experience of any observer (as opposed to a theory describing correlations between points remaining forever out of causal contact, making predictions which cannot be verified even in principle). But if the global point of view must be rejected in the context of black holes, why should it be retained in cosmology?

From a local point of view, eternal inflation looks quite different [62]. Let us attempt to describe only a single (though arbitrary) causally connected region. This can be defined as a “causal diamond”: the overlap between the causal future and the causal past of a worldline [10]. As seen in Fig. 5, this restriction eliminates most of the global spacetime. In particular, eternal inflation is no longer eternal.

Consider a geodesic worldline, starting in some initial vacuum o with large positive

cosmological constant. (Really, I am considering an ensemble of worldlines and regions causally connected to them, in the sense usually adopted to give meaning to probabilities in quantum mechanics: identical copies of a system. I am *not* demanding that the members of this ensemble coordinate their evolution so as to fit together and form a well-defined global spacetime.) Since the probability to do so is nonzero, the worldline eventually enters a vacuum of zero or negative cosmological constant, from which it will decay no further.¹⁴ But which vacua the worldline passes through, on its way to a “terminal” vacuum, is a matter of probability.

The probability for the worldline to enter vacuum i , p_i , is proportional to the expected number of times it will enter vacuum i . This can be computed straightforwardly, and unambiguously, from the matrix of transition rates between vacua [61].

The probabilities p_i depend on the initial probability distribution for the vacuum in which the worldline starts out, as one would expect in most dynamical systems. Inflation does not remove the need for a theory of initial conditions. I will not address this question here, except to say that I find it plausible that the universe began in a vacuum with large cosmological constant, and was equally likely to start in any such vacuum. The vast majority of vacua will have large cosmological constant, so this is not a strong assumption.

The resulting probability measure is predictive. In the semiclassical regime, decays tend to be exponentially suppressed, so that one decay channel typically dominates completely in any given vacuum. One would expect that a number of decays have to happen before the worldline enters a vacuum on the Weinberg shell, and that the fast decays happen first. For example, in a model of the type described in Sec. 4.3, the production of a membrane of type i is less suppressed if the background has more than one unit of the corresponding flux ($n_i > 1$), or if the charge q_i associated with the membrane is relatively small. One thus predicts that the number of units of flux should be 0 or 1 for most fluxes in our vacuum, and that we are unlikely to find fluxes associated to small charges turned on [68].

The paradox of Ref. [56] is resolved because the size of the causal diamond is cut off by the cosmological constant. It will never become larger than the horizon in a given vacuum, no matter how much slow-roll inflation occurs after the corresponding bubble is formed. Thus, exponentially large expansion factors do not enter. This does not mean that the measure is insensitive to the important question of whether inflation occurs. However, that issue arises only if we ask about the suitability of vacua for observers. I will turn to this question next.

¹⁴If Λ vanishes exactly then the vacuum is presumably supersymmetric and stable. If $\Lambda < 0$ the open universe collapses in a big crunch after a time of order $\Lambda^{-1/2}$, which is likely to be faster than any further decay channels.

5.3 Beyond the anthropic principle

Most vacua will not contain observers. This statement is not particularly controversial: for example, most vacua will have a cosmological constant of order unity, and hence will not give rise to causally connected regions much larger than a Planck length. Entropy bounds [69, 70] imply that such regions contain at most a few degrees of freedom, and only a few bits of information. This rules out complex structures.

Therefore, the probability for a worldline to enter a given vacuum, p_i , is not the same thing as the probability for that vacuum to be observed, π_i . Let us define a weight w_i that measures (in a sense to be quantified below) the chance that the vacuum i contains observers. Then

$$\pi_i = \frac{p_i w_i}{\sum p_j w_j} . \quad (5.1)$$

Estimating the weights w_i is awkward for a number of reasons. The biggest difficulty is to define what we mean by an “observer”. And given a definition, it can still be extremely hard to estimate whether observers will form in a given vacuum. What we can do reasonably well is to consider hypothetical, small changes of one or two of the parameters describing our own vacuum, and compute their effect on the formation of life like ours. But this is of little use for estimating the weights w_i of other vacua in the landscape, since they generically have radically different low-energy physics. Some correlations may appear quite robust, such as Weinberg’s assertion that some kind of structure formation is a prerequisite for observers. But others seem hopelessly specific. For example, can we seriously expect that life requires carbon? What would this statement even mean in a low-energy theory with a different standard model gauge group?

In the global approach, an additional difficulty arises: Strictly, w_i is either 0 or 1. Either there are observers in vacuum i , or there are not. Intuitively, this seems too crude; there should be a more nuanced sense in which some vacua can be more or less hospitable to life. But how would we tell whether a vacuum contains more observers than another? Each bubble is an infinite homogeneous open universe. At all times, the spatial volume is strictly infinite. So if observers can form at all, there will be an infinite number of them. (A method for dealing with this problem within the global approach has been suggested in Ref. [53].)

In the local approach, this problem does not arise. The causal diamond will be at most of linear size $|\Lambda_i|^{-1/2}$, where Λ_i is the cosmological constant in vacuum i . (I will ignore vacua with vanishing cosmological constant, since they would have to be exactly supersymmetric, ruling them out as hosts of complex structures.) Thus, the causally connected region is automatically finite, providing a natural cutoff.

The local approach can also help overcome the problem of the excessive specificity of anthropic considerations [61]. The key idea is that observers, whatever they may consist of, need to be able to increase the entropy. It is implausible that complex systems like observers will still operate when everything has thermalized and all free energy has been used up. Everything interesting happens while the universe returns to equilibrium after the phase transition associated with the formation of a new bubble.¹⁵

Let us assume that every binary operation will increase the entropy by at least an amount of order unity [71]. On average, one would expect the number of observers to be related to the total amount by which entropy increases in a given vacuum. Of course, in the global viewpoint this statement would be nonsense: if the entropy increases at all, it will increase by an infinite amount over the infinite open space. In the local viewpoint, the entropy increase is not only finite but can be very sharply defined in terms of the causal diamonds themselves.

The entropy increase is the difference between the entropy entering the diamond through the bottom cone, S_{in} , and the entropy leaving through the top cone, S_{out} , as shown in Fig. 6:

$$\Delta S = S_{\text{out}} - S_{\text{in}} . \quad (5.2)$$

The proposal is to weight each vacuum by the entropy increase it admits

$$w_i = \Delta S(i) . \quad (5.3)$$

Two observers will increase the entropy twice as much as one, so I have chosen a linear weighting. (There may be nonlinear effects, for example a sharp cutoff on the minimum entropy increase required to have at least one observer; smaller ΔS would be assigned weight zero.)

To be precise, let us take the tip of the bottom cone to lie on the reheating surface (if there is one; otherwise, no entropy is produced in any case). Before this time, the universe is empty, because bubble formation is strongly suppressed (Sec. 4.3). Only after reheating will there be matter, and it can organize itself no faster than at the speed of light. The tip of the top cone can be taken to be at a very late time, or even after the vacuum decays; in the late-time limit the entropy S_{out} will converge quickly. In a vacuum with positive cosmological constant, the top cone will coincide with the de Sitter horizon.

I will not include entropy associated with event horizons. This would dominate, particularly through the contribution from the cosmological horizon in de Sitter space. Unlike matter entropy production, it is not clear how an increase in Bekenstein-Hawking entropy can be related to the physical process of observation. However, one could consider including this contribution for formal simplicity. In this case, the argument

¹⁵This is the reason why I defined the p_i to be the probability for the worldline to *enter* vacuum i , rather than the expected amount of time the worldline will spend in vacuum i . The latter will typically be exponentially greater than the thermalization time scale and hence is of no relevance.

for prior-based predictions below would need to be augmented by the extra assumption that in *our* vacuum, the entropy produced when black holes are formed, or when they evaporate, is not related to observers. The prior-free prediction of $-\log \Lambda$ below would be strengthened, on the other hand, since the horizon entropy is inversely proportional to Λ .

Let us compare this “entropic” weighting to the anthropic principle. The latter has been used to predict quantities (such as the cosmological constant) based on other parameters of our particular vacuum (such as the time of galaxy formation). In fact it has *only* been used to make such “prior-based” predictions. Other examples (some of which happened to be post-dictions) include bounds on the density contrast $\delta\rho/\rho$ [60] and on curvature [72, 73].

In this relatively modest arena, the entropic weighting competes very well [63]. It turns out that the entropy increase of our own vacuum is dominated by the photons produced by stars, giving $\Delta S \approx 10^{85}$. This means that any variation of parameters that interferes with star formation will cause ΔS to drop drastically. For example, if Λ were much larger, no structure would form, and hence no stars would form, so this possibility is suppressed by a large drop in ΔS . In this way, the entropic weighting reproduces the successes of the anthropic principle in bounding Λ , $\delta\rho/\rho$, and curvature in terms of observed priors.

This success is remarkable. The assumptions going into anthropic arguments are quite specific and detailed. By contrast, the entropic weighting is based on a single, simple thermodynamic condition that observers must satisfy: they must be able to increase the entropy.

In some cases, the entropic weighting will even lead to better quantitative agreement between predictions and data. Anthropic arguments still expect the cosmological constant to be about 100 times larger than observed [24, 74, 75]. Large values of Λ are preferred because there are more such vacua, and the anthropic cutoff is somewhat above the observed value. In the entropic weighting, the preference for large Λ is weaker: The overall mass included in the causal diamond scales like $\Lambda^{-1/2}$. This shifts the preferred value to smaller Λ , in better agreement with observation.

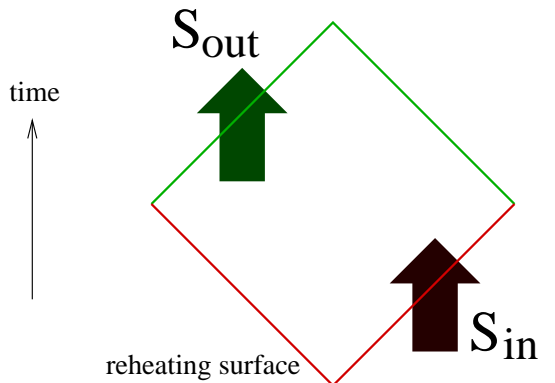


Figure 6: Instead of explicit anthropic requirements, a new proposal is to weight each vacuum by the amount of entropy, ΔS , produced after reheating. This is the difference between the entropy entering the bottom cone of the causal diamond, S_{in} , and the entropy going out through the top cone, S_{out} .

Entropic weighting may allow us to attempt predictions *without* priors, a feat thoroughly beyond the ambition of anthropic reasoning. For example, one might ask where a scale like 10^{-123} ultimately comes from [13]. Anthropic arguments only relate the cosmological constant in our vacuum to the time of galaxy formation in our vacuum. But in some other vacuum, perhaps stars could have formed much earlier, allowing the cosmological constant to be much larger [76].

In fact, this is a serious concern. In the string landscape, many parameters vary, including Λ , but also $\delta\rho/\rho$, the baryon-to-photon ratio, etc. Taking this into account, is the small observed value of Λ not terribly unlikely after all? Weinberg showed only that Λ could not be much larger *if all other parameters are held fixed*. But they are not, and this may spoil his explanation of the smallness of the cosmological constant. (It cannot spoil his prediction, which, quite sensibly, took observed data into account. But it could shift the mystery to questions such as why $\delta\rho/\rho$ or the baryon-to-photon ratio are so small.)

To address this issue, let us define a weight that depends only on Λ , with individual vacua "integrated out":

$$W(\Lambda)d\Lambda = \sum w_i = \sum \Delta S(i) , \quad (5.4)$$

where i runs over all the vacua with cosmological constant between Λ and $\Lambda + d\Lambda$. Here $d\Lambda$ should be chosen large enough for the sum to include a large number of vacua. Thus $W(\Lambda)$ is an average weight as a function of Λ .

The individual weights in this sum will vary hugely. In fact, I would expect that $\Delta S(i)$ will typically be quite small. That is, it should be atypical to get inflation and reheating, let alone to dynamically develop complex processes that produce a lot of entropy after reheating. But we are interested only in the average of the weights w_i when summing over a lot of vacua, and in fact we only care how this average depends on Λ .

Let us now make an assumption: suppose that the average is proportional to (though perhaps much smaller than) the maximum weight a vacuum can theoretically have, given Λ . The entropy difference cannot be greater than the entropy S_{out} . This is turn is bounded by the second law of thermodynamics: it must not exceed the entropy of the cosmological horizon, which is $3\pi/\Lambda$. (I am not counting horizon entropy towards ΔS , since it seems unrelated to the probability of observers, but it can still be used to bound the entropy produced by matter.)

In fact this bound can be saturated: the total mass inside the horizon can be up to $\Lambda^{-1/2}$, and the lowest energy quanta one can burn it into have wavelength $\Lambda^{-1/2}$, so one can produce up to $1/\Lambda$ quanta. Of course, one would not expect this extreme limit to be attained in any significant fraction of vacua (in our own we are down by 10^{-38}).

The idea is just that the average weight should scale in the same way with Λ as the maximum weight. So this gives

$$W(\Lambda) \propto \Lambda^{-1} . \quad (5.5)$$

Neglecting for a moment the finiteness of the discretuum density, the probability for Λ to be between a and b will thus be proportional to $\log a - \log b$.

Now let us assume a discretuum of vacua with roughly even spacing $1/N$, and $N \approx 10^{500}$. Thus $\log \Lambda$ will range from -500 to 0 . According to the above probability, observers should find themselves at some generic place in this interval, i.e., $-\log \Lambda$ should be $O(100)$.

Clearly, the assumptions going into this argument warrant further investigation. Moreover, the result is far less precise than the Weinberg prediction. This was to be expected when all recourse to previously measured quantities is abandoned. But it is reassuring that quite conceivably, the observed value of the cosmological constant does not become enormously unlikely, even if all other parameters are allowed to scan; in fact it remains quite typical.

More generally, the argument illustrates that even in the landscape, we need not give up on predicting observable parameters from the fundamental theory. Under the stated assumptions, the order of magnitude of the logarithm of the size of the universe is related to the topological complexity of six-dimensional compact manifolds. This result is prior free in the sense that it does not use properties of any particular vacuum, just the structure of the theory.

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